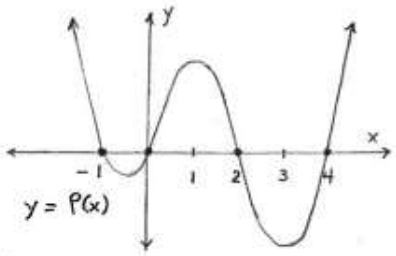
Section 4.7

Section 4.7

(86) Solve the polynomial inequality $P(x) < 0$.



86

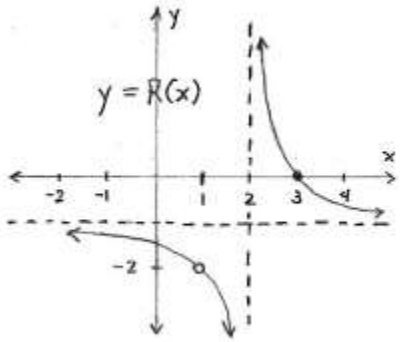
The graph is below the x-axis when $-1 < x < 0$ or $2 < x < 4$

Answer: $(-1,0) \cup (2,4)$

In Exercises 87-88, solve the rational inequality $R(x) < 0$.

87

(87)

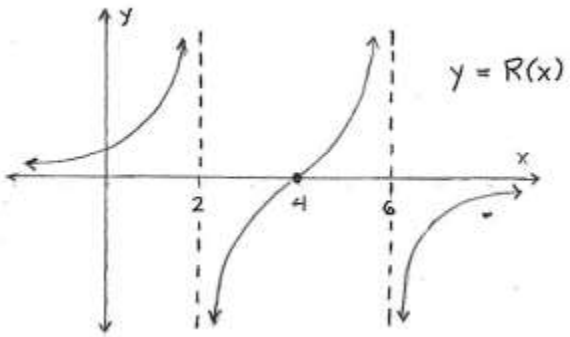


The graph has a hole at x = 1 and vertical asymptote at x = 2 so we exclude these values in the solution.
The graph is below the x-axis when $x < 1$ or $1 < x < 2$ or $x > 3$

Answer: $(-\infty, 1) \cup (1,2) \cup (3, \infty)$

88

(88)



The vertical asymptotes are x = 2 and x = 6 so we exclude these values in the solution
The graph is below the x-axis when 2 < x < 4 or x > 6

Answer: $(2,4) \cup (6,\infty)$

89

(89) Solve $x^4 - 26x^2 + 25 \geq 0$ (a) symbolically and (b) graphically.

a) Symbolically

First, solve for the boundary numbers

$x^4 - 26x^2 + 25 = 0$

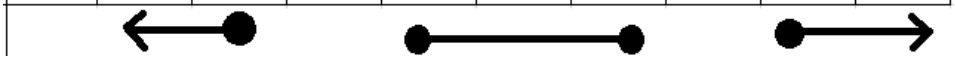
$(x^2 - 1)(x^2 - 25) = 0$

$(x - 1)(x + 1)(x - 5)(x + 5) = 0$

$x = 1, -1, 5, -5$

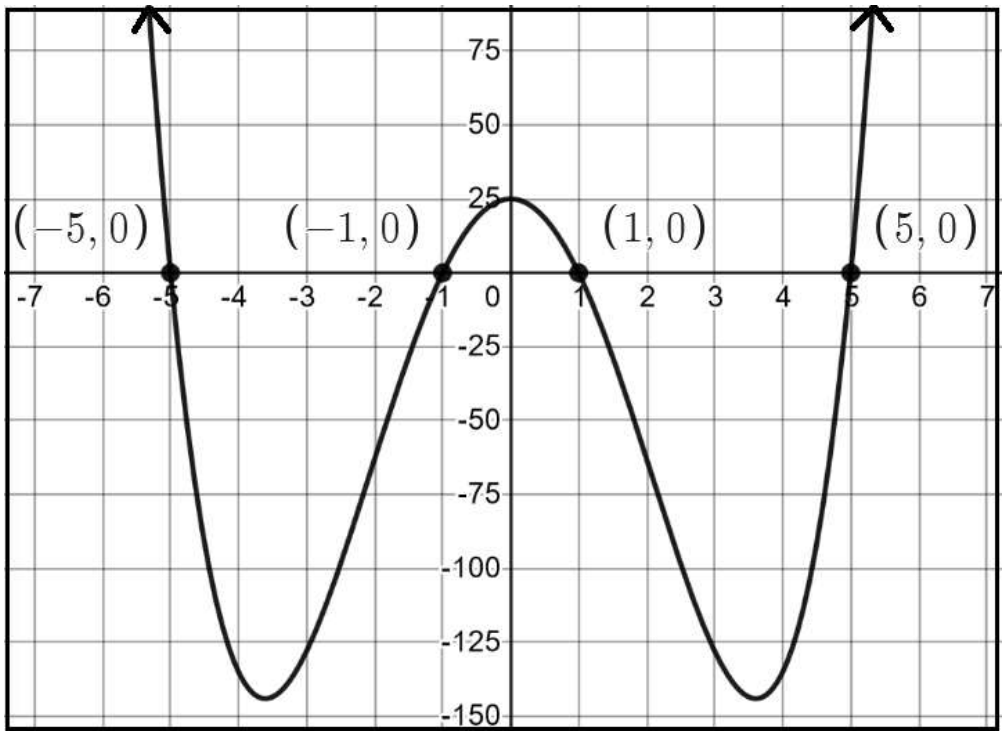
Table of Values:

x	-6	-5	-2	-1	0	1	2	5	6
x-5	-		-		-		-		+
x-1	-		-		-		+		+
x+1	-		-		+		+		+
x+5	-		+		+		+		+
f(x)	+	0	-	0	+	0	-	0	+



Answer: $(-\infty, -5] \cup [-1,1] \cup [5,\infty)$

b) Graphically



The graph is on or above the x-axis when $x \leq -5$ or $-1 \leq x \leq 1$ or $x \geq 5$

Answer: $(-\infty, -5] \cup [-1,1] \cup [5, \infty)$

90

(90) Solve $\frac{2x-5}{x^2-1} \leq 0$ symbolically.

First, solve for the boundary numbers

$2x - 5 = 0$ or $x^2 - 1 = 0$

$x = 5/2$ or $(x - 1)(x + 1) = 0$

The boundary numbers are -1, 1, and 5/2

Table of Values:

x	-2	-1	0	1	2	5/2	3
2x-5	-	-	-	-	-		+
x-1	-	-	-		+		+
x+1	-		+		+		+
f(x)	-	undefined	+	undefined	-	0	+



Answer: $(-\infty, -1) \cup \left(1, \frac{5}{2}\right]$

Section 4.8

Section 4.8

(91) Evaluate by hand.

(a) $125^{2/3}$ (b) $256^{-3/4}$ (c) $(-32)^{4/5}$

a) $125^{2/3} = \left(125^{1/3}\right)^2 = \left(\sqrt[3]{125}\right)^2 = 5^2 = \mathbf{25}$

b) $256^{-3/4} = \frac{1}{256^{3/4}} = \frac{1}{\left(256^{1/4}\right)^3} = \frac{1}{\left(\sqrt[4]{256}\right)^3} = \frac{1}{4^3} = \frac{1}{64}$

c) $(-32)^{4/5} = \left((-32)^{1/5}\right)^4 = \left(\sqrt[5]{-32}\right)^4 = (-2)^4 = 16$

92

(92) Convert to fractional exponents and simplify.

(a) $\sqrt[6]{x^5}$ (b) $\sqrt{x^3} \cdot \sqrt[3]{x}$ (c) $\sqrt{x \cdot \sqrt[4]{x}}$

a) $\sqrt[6]{x^5} = x^{5/6}$

b) $\sqrt{x^3} \cdot \sqrt[3]{x} = x^{3/2} \cdot x^{1/3} = x^{\left(\frac{3}{2}+\frac{1}{3}\right)} = x^{11/6}$

c) $\sqrt{x \cdot \sqrt[4]{x}} = \left(x \cdot x^{1/4}\right)^{1/2} = x^{1/2} \cdot x^{1/8} = x^{\left(\frac{1}{2}+\frac{1}{8}\right)} = x^{5/8}$

93

(93) Solve the radical equation.

(a) $x^{3/2} = 125$ (b) $x^{-4/3} = 256$ (c) $(x-10)^{2/3} = 25$

a) $x = 125^{2/3} = \left(125^{1/3}\right)^2 = \left(\sqrt[3]{125}\right)^2 = 5^2 = \mathbf{25}$

b) $x = 256^{-3/4} = \frac{1}{256^{3/4}} = \frac{1}{\left(256^{1/4}\right)^3} = \frac{1}{\left(\sqrt[4]{256}\right)^3} = \frac{1}{4^3} = \frac{1}{64}$

c) $x - 10 = 25^{3/2} = \left(25^{1/2}\right)^3 = \left(\sqrt{25}\right)^3 = 5^3 = 125$

Therefore, add 10 to both sides of $x - 10 = 125$

x = **135**

94

(94) Solve: $4n^{-2} - 7n^{-1} - 2 = 0$.

Let $y = n^{-1} = \frac{1}{n}$

$4y^2 - 7y - 2 = 0$

$(4y + 1)(y - 2) = 0$

$4y + 1 = 0$ or $y = 2$

$y = -1/4$ or $y = 2$

$\frac{1}{n} = -1/4$ or $\frac{1}{n} = 2$

Get the reciprocal of both sides of each equation

$n = -4$ or $n = 1/2$

95

(95) Solve: (a) $x^{4/3} - 13x^{2/3} + 36 = 0$

(b) $x^{4/5} - 3x^{3/5} - 10 = 0$. Round to 2 decimal places.

Part-a)

Let $y = x^{2/3}$

$y^2 - 13y + 36 = 0$

$(y - 4)(y - 9) = 0$

$y - 4 = 0$ or $y - 9 = 0$

$y = 4$ or $y = 9$

$x^{2/3} = 4$ or $x^{2/3} = 9$

$x = 4^{3/2}$ or $x = 9^{3/2}$

$x = \left(4^{1/2}\right)^3$ or $x = \left(9^{1/2}\right)^3$

$x = \left(\sqrt{4}\right)^3$ or $x = \left(\sqrt{9}\right)^3$

$x = 2^3$ or $x = 3^3$

$x = 8$ or $x = 27$

Part-b)

Let $y = x^{3/5}$

$y^2 - 3y - 10 = 0$

$(y + 2)(y - 5) = 0$

$y + 2 = 0$ or $y - 5 = 0$

$y = -2$ or $y = 5$

$x^{3/5} = -2$ or $x^{3/5} = 5$

$x = (-2)^{5/3}$ or $x = 5^{5/3}$

$x = \sqrt[3]{(-2)^5}$ or $x = \sqrt[3]{5^5}$

$x = \sqrt[3]{-32}$ or $x = \sqrt[3]{3125}$

$x \approx -3.17$ or $x \approx 14.62$